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of years between the epoch and that later opposition from which the epoch had been calculated. For instance, instead of *Mars* being in opposition on about the 22nd of March, 1866, a date estimated from the opposition of April 2nd, 1856, the Earth was

$$50''{\cdot}1 \times 190 = 9919'' = 2\tfrac{3}{4}^{\circ} \text{ (about)}$$

in advance of *Mars*. This affects the correction for geocentric longitude in the comparison between Hooke's observation and Dawes'. The planet being, on March 13th, 1666, nearly in opposition, and about $\frac{2}{3}$rds as far from the Earth as the Earth from the Sun, we obtain (roughly) a correction of

$$2\tfrac{3}{4}^{\circ} \times \frac{3}{2} = 4\tfrac{1}{8}^{\circ} \text{ about}$$

This correction is *subtractive*, and corresponds to rather more than $\frac{1}{90}$th of a Martial day, divided by the number of rotations in about 190 years, that is, to rather more than

$$\left(\frac{88642{\cdot}3}{90} \div 67650 \text{ seconds}\right)$$

or as nearly as possible to 0·015 seconds, or diminishes the rotation-period to

$$24^{\text{h}}\ 37^{\text{m}}\ 22^{\text{s}}{\cdot}745,$$

which we cannot consider accurate beyond the second decimal place. I believe the true value lies certainly between

$$24^{\text{h}}\ 37^{\text{m}}\ 22^{\text{s}}{\cdot}74$$

and

$$24^{\text{h}}\ 37^{\text{m}}\ 22^{\text{s}}{\cdot}75.$$

It will of course be obvious that, since the whole correction due to the error here detected is only one-seventieth of a second, it would have been useless to have calculated the correction more closely. Also the correction affects all the determinations by about the same amount.

Another effect of the correction is that, instead of 79 sidereal revolutions of the Earth and 42 of *Mars* differing by less than one day, as stated above, the true difference is 2·116 days.

On an Expression for the Angular Distance of two Planets.

By A. Cayley, Esq.

If for the planet m, referred to any fixed plane and origin of longitudes, we have

v, the longitude in orbit
θ, the longitude of node
ϕ, the inclination

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and similarly for the planet m' referred to the same fixed plane and origin of longitudes, if the corresponding quantities are v', θ', ϕ'; then the angular distance of the two planets will of course be expressible in terms of v, θ, ϕ, v', θ', ϕ', but I am not aware that the actual expression has been given. To obtain it in the most simple manner, I write further for the planet m:—

$\theta + x$, the reduced longitude,
y, the latitude,
z, the distance from node,

so that z $(= v - \theta)$, x, y, are the hypothenuse, base, and perpendicular of a right-angled spherical triangle, the base angle of which is $= \phi$. And similarly $\theta' + x'$, y', z', have the like significations for the planet m'. I write also r, r', for the distances of the two planets respectively.

This being so, the rectangular co-ordinates of the planet m are

$$\begin{aligned}&r \cos y \cos(\theta + x),\\ &r \cos y \sin(\theta + x),\\ &r \sin y,\end{aligned}$$

But observing that from the right-angled triangle we have

$$\begin{aligned}\cos z &= \cos x \cos y,\\ \cos \phi &= \tan x \cot z,\\ \sin x &= \cot \phi \tan y;\\ \sin y &= \sin \phi \sin z,\end{aligned}$$

and therefore also

$$\sin x \cos y = \cot \phi \sin y = \cos \phi \sin z,$$

the expressions for the co-ordinates become

$$\begin{aligned}&r(\cos z \cos \theta - \sin z \sin \theta \cos \phi),\\ &r(\cos z \sin \theta + \sin z \cos \theta \cos \phi),\\ &r(\qquad\qquad \sin z \sin \phi).\end{aligned}$$

Forming the analogous expressions for the co-ordinates of m', then if H be the angular distance of the two planets, we deduce at once the expression for cos H, viz. this is—

$$\begin{aligned}\operatorname{Cos} H = {}&(\cos z \cos \theta - \sin z \sin \theta \cos \phi)(\cos z' \cos \theta' - \sin z' \sin \theta' \cos \phi')\\ &+ (\cos z \sin \theta + \sin z \cos \theta \cos \phi)(\cos z' \sin \theta' + \sin z' \cos \theta' \cos \phi')\\ &+ (\qquad \sin z \sin \phi \qquad)(\qquad \sin z' \sin \phi' \qquad),\end{aligned}$$

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or, multiplying out, this is

$$\begin{aligned}\text{Cos H} &= \cos z \cos z' \cos(\theta-\theta') \\ &+ \cos z \sin z' \sin(\theta-\theta') \cos\phi' \\ &- \sin z \cos z' \sin(\theta-\theta') \cos\phi \\ &+ \sin z \sin z' (\cos(\theta-\theta') \cos\phi \cos\phi' + \sin\phi \sin\phi'),\end{aligned}$$

say this is

$$\begin{aligned}&= \text{A} \cos z \cos z' \\ &+ \text{B} \cos z \sin z' \\ &+ \text{C} \sin z \cos z' \\ &+ \text{D} \sin z \sin z',\end{aligned}$$

viz. it is

$$\begin{aligned}&= \cos(z-z') \,.\, \tfrac{1}{2}\text{A} + \tfrac{1}{2}\text{D} \\ &+ \sin(z-z') \,.\, -\tfrac{1}{2}\text{B} + \tfrac{1}{2}\text{C} \\ &+ \cos(z+z') \,.\, \tfrac{1}{2}\text{A} - \tfrac{1}{2}\text{D} \\ &+ \sin(z+z') \,.\, \tfrac{1}{2}\text{B} + \tfrac{1}{2}\text{C}\end{aligned}$$

But we have

$$z - z' = v - v' - \theta + \theta', \quad z + z' = v + v' - \theta - \theta',$$

whence the expression becomes

$$\begin{aligned}\text{Cos H} = &\cos(v-v') \,.\, \left(\tfrac{1}{2}\text{A} + \tfrac{1}{2}\text{D}\right)\cos(\theta-\theta') - \left(-\tfrac{1}{2}\text{B} + \tfrac{1}{2}\text{C}\right)\sin(\theta-\theta') \\ &+ \sin(v-v') \,.\, \left(\tfrac{1}{2}\text{A} + \tfrac{1}{2}\text{D}\right)\sin(\theta-\theta') + \left(-\tfrac{1}{2}\text{B} + \tfrac{1}{2}\text{C}\right)\cos(\theta-\theta') \\ &+ \cos(v+v') \,.\, \left(\tfrac{1}{2}\text{A} - \tfrac{1}{2}\text{D}\right)\cos(\theta+\theta') - \left(\tfrac{1}{2}\text{B} + \tfrac{1}{2}\text{C}\right)\sin(\theta+\theta') \\ &+ \sin(v+v') \,.\, \left(\tfrac{1}{2}\text{A} - \tfrac{1}{2}\text{D}\right)\sin(\theta+\theta') + \left(\tfrac{1}{2}\text{B} + \tfrac{1}{2}\text{C}\right)\cos(\theta+\theta'),\end{aligned}$$

or substituting for A, B, C, D, their values, and after a few easy reductions, we find

$$\text{Cos H} = \cos(v-v') \left\{ \begin{array}{ll} \tfrac{1}{2} + \tfrac{1}{2}\cos\phi\cos\phi' - \tfrac{1}{2}(1-\cos\phi)(1-\cos\phi') & \sin^2(\theta-\theta') \\ \qquad\qquad\qquad + \tfrac{1}{2}\sin\phi\sin\phi' & \cos(\theta-\theta') \end{array} \right\}$$

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$$
\begin{aligned}
&+ \sin(v-v') \left\{ \begin{aligned} &\tfrac{1}{2}(1-\cos\varphi)(1-\cos\varphi') \sin(\theta-\theta') \cos(\theta-\theta') \\ &+ \tfrac{1}{2} \sin\varphi \sin\varphi' \qquad\qquad \sin(\theta-\theta') \end{aligned} \right\} \\
&+ \cos(v+v') \left\{ \begin{aligned} &\tfrac{1}{2}(1-\cos\varphi\cos\varphi') \cos(\theta-\theta') \cos(\theta+\theta') \\ &+ \tfrac{1}{2}(\cos\varphi-\cos\varphi') \quad \sin(\theta-\theta') \sin(\theta+\theta') \\ &- \tfrac{1}{2} \sin\varphi \sin\varphi' \qquad\qquad \cos(\theta+\theta') \end{aligned} \right\} \\
&+ \sin(v+v') \left\{ \begin{aligned} &\tfrac{1}{2}(1-\cos\varphi\cos\varphi') \cos(\theta-\theta') \sin(\theta+\theta') \\ &- \tfrac{1}{2}(\cos\varphi-\cos\varphi') \sin(\theta-\theta') \cos(\theta+\theta') \\ &- \tfrac{1}{2} \sin\varphi \sin\varphi' \qquad\qquad \sin(\theta+\theta') \end{aligned} \right\}.
\end{aligned}
$$

For $\varphi = \varphi' = 0$, the formula becomes, as of course it should do,

$$\text{Cos H} = \cos(v - v').$$

It may be added, that if f, f' are the true anomalies, ω, ω' the longitudes of pericentre in orbit, then $v = \omega + f$, $v' = \omega' + f'$; and we thence have for cos H, formulæ of the like form, containing $\cos f \cos f'$, $\cos f \sin f'$, $\sin f \cos f'$, $\sin f \sin f'$, or containing $\cos(f-f')$, $\sin(f-f')$, $\cos(f+f')$, $\sin(f+f')$ respectively, in place of the like functions of z, z', but with of course altered values of the co-efficients.

Note on Stars within the Trapezium of the Nebula in Orion. By David Gill, Jun., Esq.

On the 22nd February, 1867, at 10 p.m., G.M.T., I had been measuring the position and distances of two components of the multiple star θ *Orionis*. The definition being unusually fine, I extinguished the lamp to look at the Nebula.

I was immediately struck by a twinkling within the trapezium which, by careful watching, I was enabled to map down as two faint stars.

On receiving the *Astr. Register* for March 1867, I found that Mr. Huggins' diagram there given, corresponded with mine as to the stars 5 and 7. This diagram is copied from the *Monthly Notices* for January 1866.

My diagram also showed a star marked "suspected," similar in R.A. to No. 9, and of the Decl. of δ.

This star was so very, very faint I could only catch it by momentary glimpses, and can hardly depend on its position.